hands of my friend and colleague, Professor Roscoe, F.R.S., the only sample available of the protagon of which the analyses had been communicated to the Royal Society; this was a twice re-crystallized sample of protagon from ox-brain, which had been employed for analyses 3 and 4. The remaining specimens had been used in experiments on the products of decomposition of protagon.

I requested Dr. Roscoe, to whom I communicated Dr. Thudichum's criticism, to determine the amount of mineral impurities, and especially of potassium present in protagon. He has kindly furnished me with the ensuing report, with which I shall close this communication, as I am convinced that no one will expect me to enter into a polemical discussion with the author of the previously quoted matter.

The Owens College, Manchester,
December, 1879.

I have examined spectroscopically for potash a sample of protagon furnished me by Dr. Gamgee, and labelled "Protagon twice re-crystallized. Blankenhorn." I could not detect any potash by the spectroscope in the incinerated mass from 0·1 grm. of substance. With the carbonized mass obtained from 1·0 grm. of substance I obtained the potassium line (α) very faintly, and from comparative experiments with a dilute solution of a potassium salt, I estimate the quantity of potash in 1 grm. of the substance not to exceed $\frac{1}{20}$ mgrm. The carbonized residue of 1 grm. of protagon was carefully oxidized with pure nitric acid, when a small quantity of fused metaphosphoric acid remained after ignition. This residue weighed 0·0278 grm., corresponding to 1·08 per cent. of phosphorus.*

(Signed) H. E. ROSCOE.

III. "On the Induction of Electric Currents in Infinite Plates and Spherical Shells." By C. NIVEN, M.A., Professor of Mathematics in Queen's College, Cork. Communicated by J. W. L. GLAISHER, M.A., F.R.S. Received January 21, 1880.

(Abstract.)

The object of the present paper is to determine the currents which are induced in an infinite plate of uniform conductivity and finite thickness, and in a sphere or spherical shell of any thickness when in the presence of a varying magnetic system: and in any of these bodies when rotating near a constant magnetic system, round an axis which is normal to the faces of the plate or passes through the centre

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* The mean quantity of phosphorus found by Gamgee and Blankenhorn was 1·068 per cent.—A. G.

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of the sphere or shell. The case of an infinitely thin plane sheet has been made the subject of a paper by the late Professor Clerk Maxwell, in the "Proceedings of the Royal Society," vol. xx (1872), and Jochmann has also treated the case of a rotating sphere when the inducing magnetism is symmetrical about the axis of rotation, and that of a rotating plate when the mutual induction of the currents is neglected. In the present paper the problems are dealt with generally.

Maxwell's equations of the field are adopted, and the peculiar view which he takes of the electric current renders it necessary to enter with some detail into the general boundary conditions. These are given, first of all, for two substances at rest, which possess both conductivity and specific dielectric capacity: we can then deduce the conditions to be satisfied at the common surface of a conductor and a dielectric.

Let F, G, H be the components, at any point just inside the surface of the conductor, of the vector potential due to all the currents and magnets in the field, $\mathfrak{J}$ the component along the normal (N) to the surface, measured outward from the conductor, ψ the electric potential; and let F′, G′, H′ be the components of the vector potential just outside, the boundary conditions are

$$F=F' \ldots \frac{dF}{dN}=\frac{dF'}{dN}, \ldots \frac{d\mathfrak{J}}{dt}+\frac{d\psi}{dN}=0.$$

When $\mathfrak{J}=0$, as in the cases treated in the paper, it is shown that ψ must vanish everywhere, and there is no free electricity within the conductor or on its surface.

Infinite Plate.

For the case of an infinite plate, the scalar potential (Ω_0) of the external system and the vector components are given by

$$\Omega_0=\frac{dP_0}{dz}, \qquad F_0=-\frac{dP_0}{dy}, \qquad G_0=\frac{dP_0}{dx}, \qquad H_0=0.$$

The vector potential of the currents in the plate, and the currents themselves are given by

$$F=-\frac{dP}{dy}, \qquad G=\frac{dP}{dx}, \qquad H=0,$$

$$u=-\frac{d\Phi}{dy}, \qquad v=\frac{d\Phi}{dx}, \qquad w=0, \text{ where } 4\pi\Phi=-\nabla^2 P.$$

They are also expressed in semipolar co-ordinates ρ, ϕ, z, thus:—

$$F=-\frac{dP}{\rho d\phi}, \quad G=\frac{dP}{d\rho}, \quad H=0, \quad u=-\frac{d\Phi}{\rho d\phi}, \quad v=\frac{d\Phi}{d\rho}, \quad w=0,$$

$$\text{where } 4\pi\Phi=-\nabla^2 P.$$

The mode of solution adopted is to consider the variations of the external system as a series of impulses, producing a succession of currents which decay according to the law

$$\frac{\sigma}{4\pi}\nabla^2 \mathrm{P}=\frac{d\mathrm{P}}{dt},\ \sigma \text{ being the conductivity.}$$

If the value of P_0 rise suddenly from 0 to P_0, the corresponding value of P will be $-\mathrm{P}_0$ initially; and, at any subsequent time, will be given, for an external point, on the positive side of the plate, by

$$\mathrm{P}=-\frac{1}{\pi}\Sigma_{m=0}^{m=\infty}\int_0^\pi \cos m\overline{\phi-\phi'}\,d\phi'\int_0^\infty \kappa\epsilon^{-\kappa(z-b)}\mathrm{J}_m(\kappa\rho)\,d\kappa\int_0^\infty \rho'\mathrm{J}_m(\kappa\rho')\,d\rho'\,.\,\mathrm{Z},$$

where

$$\mathrm{Z}=\Sigma\left\{e^{-\lambda t}\frac{2n\cos nb}{2nb+\sin 2nb}\int_{-b}^{+b}\mathrm{P}_0{}'\cos nz'dz'\right.$$

$$\left.+e^{-\lambda' t}\frac{2n'\sin n'b}{2n'b-\sin 2n'b}\int_{-b}^{+b}\mathrm{P}_0{}'\sin n'z'dz'\right\},$$

in which

$$\lambda=\frac{\sigma}{4\pi}(\kappa^2+n^2),\qquad n\sin nb-\kappa\cos nb=0.$$

$$\lambda'=\frac{\sigma}{4\pi}(\kappa^2+n'^2),\qquad n'\cos n'b+\kappa\sin n'b=0.$$

The origin is midway between the faces of the plate, and the last summation is to be extended over all the values of n and n', derived from these equations.

By putting $\mathrm{R}=\frac{\sigma}{b}$, and making b infinitely small, we deduce the solution for an infinitely thin sheet. The result coincides with Maxwell's.

Sphere or Spherical Shell.

The formulæ adapted to this case are

$$\Omega_0=\frac{d(\mathrm{P}_0 r)}{dr},\qquad \mathrm{F}_0=0,\qquad \mathrm{G}_0=-\frac{d\mathrm{P}_0}{\sin\theta d\phi},\qquad \mathrm{H}_0=\frac{d\mathrm{P}_0}{d\theta},$$

$$\mathrm{F}=0,\quad \mathrm{G}=-\frac{d\mathrm{P}}{\sin\theta d\phi},\quad \mathrm{H}=\frac{d\mathrm{P}}{d\theta},\quad u=0,\quad v=-\frac{d\Phi}{\sin\theta d\phi},\quad w=\frac{d\Phi}{d\theta},$$

and

$$4\pi\Phi=-\nabla^2\mathrm{P},\qquad \frac{\sigma}{4\pi}\nabla^2\mathrm{P}=\frac{d\mathrm{P}}{dt}.$$

[We have here resolved along the elements dr, $rd\theta$, $r\sin\theta d\phi$].

When the shell is of finite thickness, the solution is expressed in terms of spherical harmonics and of the functions which are the solutions of the equation

$$\frac{d^2\mathrm{R}}{dr^2}+\frac{2}{r}\frac{d\mathrm{R}}{dr}+\left(\lambda^2-\frac{n\,.\,\overline{n+1}}{r^2}\right)\mathrm{R}=0.$$

When the shell is infinitely thin, the action of the induced currents is expressed by the following statement, which, it will be seen, is an extension of Maxwell's result for a plane sheet. Divide the time into an infinite number of equal intervals, and at the commencement of any of these let a positive image of the external system be formed at the place occupied by its electric image at the surface. Let the parts of this image move towards the centre in straight lines, so that the logarithmic decrement of their distances from the centre is constant and equal to $\frac{R}{2\pi a}$ (R being the resistance of the shell and a its radius), and let the intensity of the image increase at each point with a constant logarithmic rate $\frac{R}{4\pi a}$. At the end of the interval let an exactly equal but negative image be formed in the place of the former and move toward the centre in the same manner, and let these operations be repeated at the beginning and end of every interval: the action of the sheet on external points will be that due to the above train of images.

Induction in a Rotating Conductor.

When the conductor rotates uniformly about an axis of symmetry the currents will become, after a time, steady. In the case of an infinite plate of finite thickness, the vector potential and currents will have the same general forms as before, but the electric potential will not be zero. The solution is now expressed by taking

$$\psi=\sigma\rho\frac{d\chi}{d\rho}, \qquad \Phi=-\frac{d\chi}{d\phi}=-\frac{1}{4\pi}\nabla^2 P,$$

$$\sigma\chi=\omega(P+P_0), \qquad \frac{\sigma}{4\pi}\nabla^2 P=\frac{\omega d}{d\phi}(P+P_0),$$

(w being the angular velocity).

When the inducing magnetism lies all on the positive side of the plate, we may express P_0 within it in the form

$$\Sigma A \cos m\phi J_m(\kappa\rho)e^{\kappa z},$$

the origin being on the positive face. The value of P at any external point on the positive side may be expressed by

$$P=\Sigma C \cos m\phi J_m(\kappa\rho)e^{-\kappa z},$$

where $$\frac{C}{A}=\frac{\kappa^2-\mu^2}{2\kappa}\cdot\frac{e^{\mu b}-e^{-\mu b}}{\kappa(e^{\mu b}+e^{-\mu b})-\mu(e^{\mu b}-e^{-b})}, \qquad \mu^2=\kappa^2-\frac{2\pi m\omega}{\sigma}\,.\,i.$$

When the plate becomes a thin sheet we may put $\sigma = \mathrm{R}b$, and make b infinitely small; we shall then have

$$\mathrm{R}\frac{d\mathrm{P}}{dz} - 2\pi\omega\frac{d\mathrm{P}}{d\phi} = 2\pi\omega\frac{d\mathrm{P}_0}{d\phi}.$$

If we express P and P_0 as functions of ρ, ϕ, $z + \frac{\mathrm{R}}{2\pi\omega}\phi$,

$$\mathrm{P} = -\int\left(\frac{d\mathrm{P}_0}{d\phi}\right)d\phi,$$

$\left(\frac{d\mathrm{P}_0}{d\phi}\right)$ being found on the original system of variables. There seems to be a mistake in Maxwell's integration here, but it does not affect the representation of the result by means of the spiral trail of images which he has given.

Spherical Conductor.

In the case of a sphere or spherical shell, the vector potentials and currents will have the same general forms as before, and the solution is contained in the equations

$$\psi = \sigma r \sin\theta\frac{d\chi}{d\theta}, \qquad \Phi = -\frac{d\chi}{d\phi} = -\frac{1}{4\pi}\nabla^2\mathrm{P},$$

$$\sigma\chi = \omega(\mathrm{P} + \mathrm{P}_0), \qquad \frac{\sigma}{4\pi}\nabla^2\mathrm{P} = \omega\frac{d}{d\phi}(\mathrm{P} + \mathrm{P}_0),$$

From the general formulæ expressing completely the effect of the shell on external points, we pass to one whose thickness (c) is infinitely small: putting then $\mathrm{R}c = \sigma$, $\mathfrak{P} = \mathrm{P}r^{\frac{1}{2}}$, $\mathrm{Q} = \mathrm{P}_0 r^{\frac{1}{2}}$, $\rho = a\log r$, we obtain finally

$$\mathrm{R}\frac{d\mathfrak{P}}{d\rho} - 2\pi\omega\frac{d\mathfrak{P}}{d\phi} = 2\pi\omega\frac{d\mathrm{Q}}{d\phi},$$

which may be integrated as before. It indicates a spiral trail of images differing somewhat from the former; that due to a single pole will form a curve lying on a cone, and would be, if the cone were developed, a series of portions of an equiangular spiral.

IV. "On the Physical Constants of Liquid Hydrochloric Acid."

By GERRARD ANSDELL, F.C.S., Chemical Assistant at the Royal Institution. Communicated by Professor DEWAR, F.R.S. Received January 22, 1880.

In a paper read before the Royal Society, on June 18th, 1879, I described the results of some experiments made with liquid acetylene, obtained by compressing the gas in one of M. Cailletet's ingenious